

A Comment on “Optimal Stroke Patterns for Purcell’s Three-Link Swimmer”

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In their letter “Optimal strokes patterns for Purcell’s three-linked swimmer” Tam and Hosoi [2] describe strokes that optimize the distance and swimming efficiency of Purcell’s three linked swimmer. The calculations are made in the framework of Cox’s slender body theory [1] where the aspect ratio κ , a.k.a. the slenderness, is large.

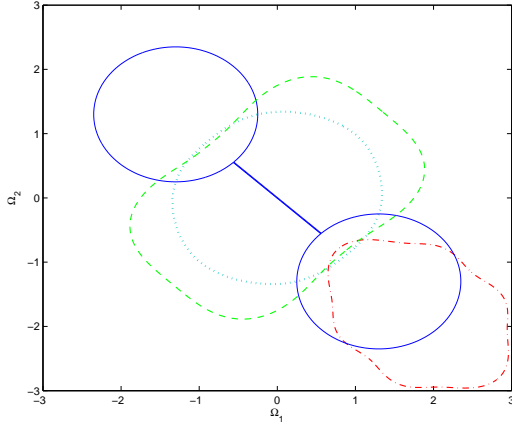


FIG. 1: Ω_1, Ω_2 are the exterior angles made by the center link and the two arms. The approximate rectangle (green) represents the distance optimizer of moderate strokes of Tam and Hosoi. The (blue) dumbbell is a large stroke that outperforms the moderate stroke. The approximate (Light-blue) circle at center represents the efficiency optimizer of moderate strokes found by Tam and Hosoi. The off-center (red) wobble oval is a large stroke with higher efficiency.

Tam and Hosoi find that the approximate square and

circle at the center of the figure are optimizers of the distance and efficiency respectively. Here we want to point out that these optimizers, with moderate strokes, are likely to be global optimizers only for κ that are moderately large. However, when κ is sufficiently large, there are large strokes that outperform the moderate optimizers. For example, the dumbbell in the figure (with a narrow “corridor” of about 0.01 rad.) will cover about 1.14 the distance covered by the approximate square, (in the *opposite* directions). Though large it is never close to self intersection (in contrast with the large stroke optimizer discussed in [3]). Cox’s slender body theory to first order is valid when the interaction forces between the links are small compared with the forces due to the motion of the links. This imposes a constraint on the angles: $(\pi - |\Omega_j|) \ln \kappa \gg 1$. In the dumbbell case where $\pi - |\Omega| \geq \frac{\pi}{4}$, the constraint is $\kappa \gg 4$. If one does not restrict Ω_j one finds that the optimizer hits the boundary, $\Omega = \pi$, and so is outside of Cox’s theory for any finite κ .

The case of optimally efficient stroke is similar. Restricting $\pi - |\Omega| \geq 0.14$ (in this case Cox’s slender body theory imposes $\kappa \gg 1250$), we have calculated the ex-center wobbly oval in the figure to be 1.53 times more efficient than the centered approximate circle of Tam and Hosoi. (Here too the optimal stroke hits the boundary $\Omega = \pi$).

It is an interesting open question to determine the minimal values of κ where the large strokes of the type described here, outperform the optimizers for moderate strokes describe in [2].

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